OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4723/01**

MATHEMATICS

Core Mathematics 3

THURSDAY 18 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Find the equation of the tangent to the curve $y = \dfrac{2x+1}{3x-1}$ at the point $\left(1, \frac{3}{2}\right)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

2 It is given that θ is the acute angle such that $\sin\theta = \frac{12}{13}$. Find the exact value of

(i) $\cot\theta$, [2]

(ii) $\cos 2\theta$. [3]

3 **(a)** It is given that a and b are positive constants. By sketching graphs of

$$y = x^5 \quad \text{and} \quad y = a - bx$$

on the same diagram, show that the equation

$$x^5 + bx - a = 0$$

has exactly one real root. [3]

(b) Use the iterative formula $x_{n+1} = \sqrt[5]{53 - 2x_n}$, with a suitable starting value, to find the real root of the equation $x^5 + 2x - 53 = 0$. Show the result of each iteration, and give the root correct to 3 decimal places. [4]

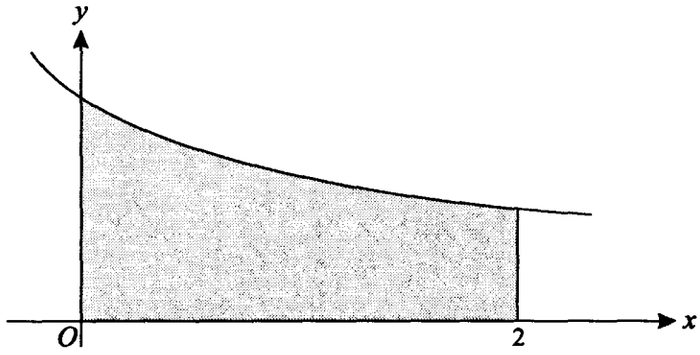
4 **(i)** Given that $x = (4t + 9)^{\frac{1}{2}}$ and $y = 6e^{\frac{1}{2}x+1}$, find expressions for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dx}$. [4]

(ii) Hence find the value of $\dfrac{dy}{dt}$ when $t = 4$, giving your answer correct to 3 significant figures. [3]

5 **(i)** Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $4\cos\theta - \sin\theta = 2$, giving all solutions for which $-180° < \theta < 180°$. [5]

6



The diagram shows the curve with equation $y = \frac{1}{\sqrt{3x+2}}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$.

(i) Find the exact area of the shaded region. [4]

(ii) The shaded region is rotated completely about the x-axis. Find the exact volume of the solid formed, simplifying your answer. [5]

7 The curve $y = \ln x$ is transformed to the curve $y = \ln\left(\frac{1}{2}x - a\right)$ by means of a translation followed by a stretch. It is given that a is a positive constant.

(i) Give full details of the translation and stretch involved. [2]

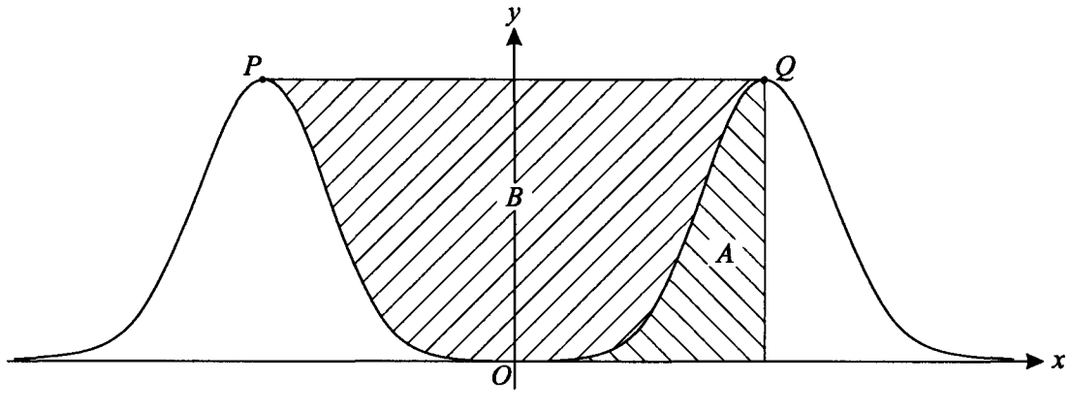
(ii) Sketch the graph of $y = \ln\left(\frac{1}{2}x - a\right)$. [2]

(iii) Sketch, on another diagram, the graph of $y = \left|\ln\left(\frac{1}{2}x - a\right)\right|$. [2]

(iv) State, in terms of a, the set of values of x for which $\left|\ln\left(\frac{1}{2}x - a\right)\right| = -\ln\left(\frac{1}{2}x - a\right)$. [2]

[Questions 8 and 9 are printed overleaf.]

8



The diagram shows the curve with equation $y = x^8e^{-x^2}$. The curve has maximum points at P and Q. The shaded region A is bounded by the curve, the line $y = 0$ and the line through Q parallel to the y-axis. The shaded region B is bounded by the curve and the line PQ.

(i) Show by differentiation that the x-coordinate of Q is 2. [5]

(ii) Use Simpson's rule with 4 strips to find an approximation to the area of region A. Give your answer correct to 3 decimal places. [4]

(iii) Deduce an approximation to the area of region B. [2]

9 Functions f and g are defined by

$$\begin{aligned} \mathrm{f}(x) &= 2\sin x \quad \text{for } -\tfrac{1}{2}\pi \leqslant x \leqslant \tfrac{1}{2}\pi, \\ \mathrm{g}(x) &= 4 - 2x^2 \quad \text{for } x \in \mathbb{R}. \end{aligned}$$

(i) State the range of f and the range of g. [2]

(ii) Show that $\mathrm{gf}(0.5) = 2.16$, correct to 3 significant figures, and explain why $\mathrm{fg}(0.5)$ is not defined. [4]

(iii) Find the set of values of x for which $\mathrm{f}^{-1}\mathrm{g}(x)$ is not defined. [6]

1 Attempt use of quotient rule to find derivative M1 allow for numerator 'wrong way round'; or attempt use of product rule

Obtain $\frac{2(3x-1)-3(2x+1)}{(3x-1)^2}$ A1 or equiv

Obtain $-\frac{5}{4}$ for gradient A1 or equiv

Attempt eqn of straight line with numerical gradient M1 obtained from their $\frac{dy}{dx}$; tangent not normal

Obtain $5x + 4y - 11 = 0$ A1 **5** or similar equiv

---

2 (i) Attempt complete method for finding $\cot\theta$ M1 rt-angled triangle, identities, calculator, …

Obtain $\frac{5}{12}$ A1 **2** or exact equiv

(ii) Attempt relevant identity for $\cos 2\theta$ M1 $\pm 2\cos^2\theta \pm 1$ or $\pm 1 \pm 2\sin^2\theta$ or $\pm(\cos^2\theta - \sin^2\theta)$

State correct identity with correct value(s) substituted A1

Obtain $-\frac{119}{169}$ A1 **3** correct answer only earns 3/3

---

3 (a) Sketch reasonable attempt at $y = x^5$ *B1 accept non-zero gradient at O but curvature to be correct in first and third quadrants

Sketch straight line with negative gradient *B1 existing at least in (part of) first quadrant

Indicate in some way single point of intersection B1 **3** dep *B1 *B1

(b) Obtain correct first iterate B1 allow if not part of subsequent iteration

Carry out process to find at least 3 iterates in all M1

Obtain at least 1 correct iterate after the first A1 allow for recovery after error; showing at least 3 d.p. in iterates

Conclude 2.175 A1 **4** answer required to precisely 3 d.p.

[0 → 2.21236 → 2.17412 → 2.17480 → 2.17479;
1 → 2.19540 → 2.17442 → 2.17480 → 2.17479;
2 → 2.17791 → 2.17473 → 2.17479 → 2.17479;
3 → 2.15983 → 2.17506 → 2.17479 → 2.17479]

---

4 (i) Obtain derivative of form $k(4t+9)^{-\frac{1}{2}}$ M1 any constant k

Obtain correct $2(4t+9)^{-\frac{1}{2}}$ A1 or (unsimplified) equiv

Obtain derivative of form $k\,e^{\frac{1}{2}x+1}$ M1 any constant k different from 6

Obtain correct $3e^{\frac{1}{2}x+1}$ A1 **4** or equiv

(ii) Either: Form product of two derivatives M1 numerical or algebraic

Substitute for t and x in product M1 using $t = 4$ and calculated value of x

Obtain 39.7 A1 **3** allow ±0.1; allow greater accuracy

Or: Obtain $k(4t+9)^n e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$ M1 differentiating $y = 6e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$

Obtain correct $6(4t+9)^{-\frac{1}{2}}e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$ A1 or equiv

Substitute $t = 4$ to obtain 39.7 A1 **(3)** allow ±0.1; allow greater accuracy

5 (i) Obtain $R = \sqrt{17}$ or 4.12 or 4.1 B1 or greater accuracy

Attempt recognisable process for finding α M1 allow for sin/cos confusion

Obtain $\alpha = 14$ A1 **3** or greater accuracy 14.036…

(ii)	Attempt to find at least one value of $\theta + \alpha$	M1		
	Obtain or imply value 61	A1√		following R value; or value rounding to 61
	Obtain 46.9	A1		allow ±0.1; allow greater accuracy
	Show correct process for obtaining second angle	M1		
	Obtain –75	A1	**5**	allow ±0.1; allow greater accuracy; max of 4/5 if extra angles between –180 and 180

---

6 **(i)**	Obtain integral of form $k(3x + 2)^{\frac{1}{2}}$	M1		any constant k
	Obtain correct $\frac{2}{3}(3x + 2)^{\frac{1}{2}}$	A1		or equiv
	Substitute limits 0 and 2 and attempt evaluation	M1		for integral of form $k(3x + 2)^n$
	Obtain $\frac{2}{3}(8^{\frac{1}{2}} - 2^{\frac{1}{2}})$	A1	**4**	or exact equiv suitably simplified
(ii)	State or imply $\pi\int \frac{1}{3x+2}\,\mathrm{d}x$ or unsimplified version	B1		allow if dx absent or wrong
	Obtain integral of form $k\ln(3x + 2)$	M1		any constant k involving π or not
	Obtain $\frac{1}{3}\pi\ln(3x + 2)$ or $\frac{1}{3}\ln(3x + 2)$	A1		
	Show correct use of ln a – ln b property	M1		
	Obtain $\frac{1}{3}\pi\ln 4$	A1	**5**	or (similarly simplified) equiv

---

7 **(i)**	State a in x-direction	B1		or clear equiv
	State factor 2 in x-direction	B1	**2**	or clear equiv
(ii)	Show (largely) increasing function crossing x-axis	M1		with correct curvature
	Show curve in first and fourth quadrants only	A1	**2**	not touching y-axis and with no maximum point; ignore intercept
(iii)	Show attempt at reflecting negative part in x-axis	M1		
	Show (more or less) correct graph	A1√	**2**	following their graph in **(ii)** and showing correct curvatures
(iv)	Identify $2a$ as asymptote or $2a + 2$ as intercept	B1		allow anywhere in question
	State $2a < x \leq 2a + 2$	B1	**2**	allow < or ≤ for each inequality

---

8 **(i)**	Obtain $-2x\mathrm{e}^{-x^2}$ as derivative of e^{-x^2}	B1		
	Attempt product rule	*M1		allow if sign errors or no chain rule
	Obtain $8x^7\mathrm{e}^{-x^2} - 2x^9\mathrm{e}^{-x^2}$	A1		or (unsimplified) equiv
	Either: Equate first derivative to zero and attempt solution	M1		dep *M; taking at least one step of solution
	Confirm 2	A1	**5**	AG
	Or: Substitute 2 into derivative and show attempt at evaluation M1			
	Obtain 0	A1	**(5)**	AG; necessary correct detail required

(ii) Attempt calculation involving attempts at y values — M1 with each of 1, 4, 2 present at least once as coefficients

Attempt $k(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$ — M1 with attempts at five y values corresponding to correct x values

Obtain $\frac{1}{6}(0 + 4 \times 0.00304 + 2 \times 0.36788 + 4 \times 2.70127 + 4.68880)$ — A1 or equiv with at least 3 d.p. or exact values

Obtain 2.707 — A1 **4** or greater accuracy; allow ±0.001

(iii) Attempt 4(y value) – 2(part **(ii)**) — M1 or equiv

Obtain 13.3 — A1 **2** or greater accuracy; allow ±0.1

---

9 (i) State $-2 \leq y \leq 2$ — B1 allow <; any notation

State $y \leq 4$ — B1 **2** allow <; any notation

(ii) Show correct process for composition — M1 right way round

Obtain or imply 0.959 and hence 2.16 — A1 AG; necessary detail required

Obtain g(0.5) = 3.5 — B1 or (unsimplified) equiv

Observe that 3.5 not in domain of f — B1 **4** or equiv

(iii) Relate quadratic expression to at least one end of range of f — M1 or equiv

Obtain both of $4 - 2x^2 < -2$ and $4 - 2x^2 > 2$ — A1 or equiv; allow any sign in each (< or ≤ or > or ≥ or =)

Obtain at least two of the x values $-\sqrt{3}, -1, 1, \sqrt{3}$ — A1

Obtain all four of the x values — A1

Attempt solution involving four x values — M1 to produce at least two sets of values

Obtain $x < -\sqrt{3}, \quad -1 < x < 1, \quad x > \sqrt{3}$ — A1 **6** allow ≤ instead of < and/or ≥ instead of >